Notes on computer simulations

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The three computer simulations, based on the use of the ODE, reported in this web page describe the time evolution of three dynamical quantities, specified cell by cell:

1. The amount of occupied residential volume in the cell (giving an estimate of the resident population)

2. An indicator able to detect high density zones with a high degree of mixing of different kind of activities.

3. An indicator able to detect lower density zones with a high degree of mixing of different kind of activities.

Some remarks are needed in order to explain the meaning of the last two indicators. Both indicators, we will call them respectively μ_{H} and μ_{L}, take values in the interval $[0,1]$ and should be meant as truth values of sentences about the degree of centrality of the cell they refer. Here “centrality” means that several kind of activities and uses of the urban space like commerce, tertiary and residences, mixes at a high degree inside the cell. The mixing of different kind of uses of the space is considered here as an indication that the considered cell takes a central place in the surrounding part of the city. This feature of the urban space is hence assumed here to be typical of centres distinguishing them by more peripheral parts of the city.
The indicator μ_{H} and μ_{L} distinguishes one form another because the first one is able to detect central zones (in the sense explained above) with a high density of built volume whereas the second one detects only small centres, with a lower degree of density of the built volume.
We point out here that these indicators aim to capture some aspects of the way an individual perceives and categorizes the urban space in which it moves and acts. In this sense these indicators should be distinguished by the ones used for urban planning in order to constraint and drive the development of the city. Centrality indicators like μ_{H} and μ_{L} and others that can be defined along similar lines can be useful to detect features of the urban space with respect to the quality of life in cities.

The indicators μ_{H} and μ_{L} have been defined using fuzzy logic method in the following way.
The first step consists in defining the ratio of the volumes $V_{\mathrm{R,C,O}}$ used inside the cell respectively for residences (R), small commercial surfaces (C) and offices (O) over the total volume $V := V_{\mathrm{R}} + V_{\mathrm{C}} + V_{\mathrm{O}}$:

$$f_{R,C,O} := \begin{cases} \dfrac{V_{R,C,O}}{V} & \text{if} \quad V \neq 0 \\ 0 & \text{if} \quad V = 0 \end{cases} \qquad (1)$$

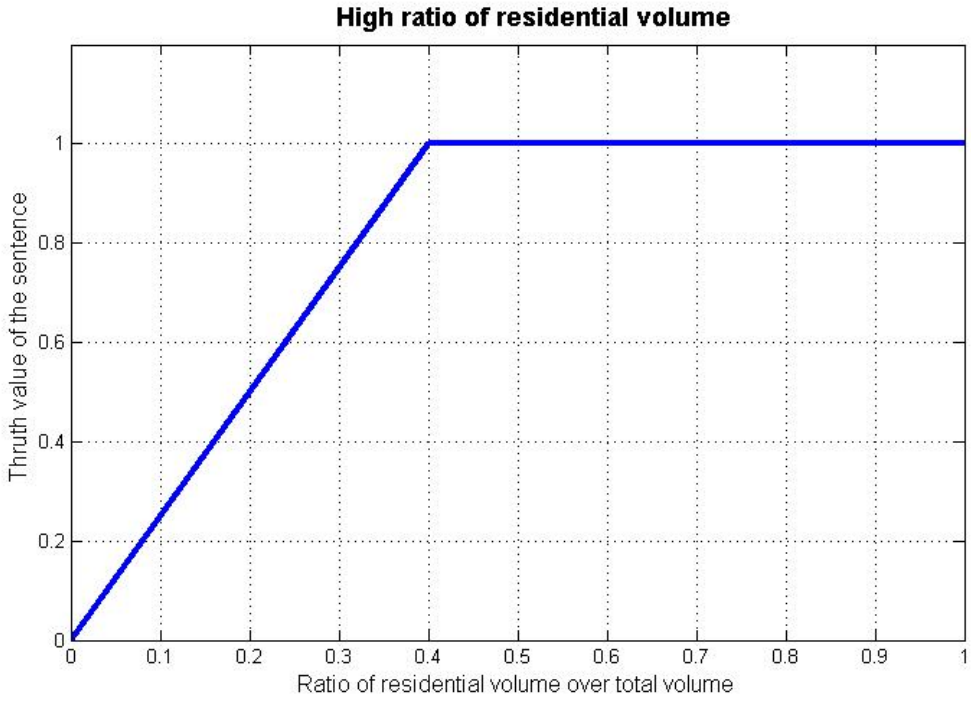
In the second step we construct the following three sentences:

$p_R \leftrightarrow$ *"There is a high ratio of residential volume over the total built volume in the cell"*

$p_C \leftrightarrow$ *"There is a high ratio of commercial volume over the total built volume in the cell"*

$p_O \leftrightarrow$ *"There is a high ratio of volume used for offices over the total built volume in the cell"*

The truth value of these sentences, indicated respectively with μ_R, μ_C and μ_O, can take values in the interval $[0,1]$ and are evaluated using typical membership functions frequently used in fuzzy set theory, like in the example above.



Different membership functions are needed for different uses, because of the different meaning of the words "high ratio" in the three cases.

A weighted mean of the truth values of μ_R, μ_C and μ_O gives the degree of mixing of the different activities:

$$\mu_M := \alpha_R \cdot \mu_R + \alpha_C \cdot \mu_C + \alpha_O \cdot \mu_O \qquad (2)$$

The quantity μ_M is interpreted as truth value of the sentence:

$p_M \leftrightarrow$ *"There is a high degree of mixing of activities inside the cell"*

where $0 \le \alpha_R, \alpha_C, \alpha_O \le 1$ and $\alpha_R + \alpha_C + \alpha_O = 1$.
In the third step a new sentence p_V is introduced with the related truth value μ_V , obtained trough a membership function of the type showed in the figure above :

$p_V \leftrightarrow$ *"There is a high amount of built volume In the cell"*

In the last step the degree of centrality μ_H is defined through a connective AND operating on the sentences p_M and p_V :

$$p_H \leftrightarrow p_H \wedge p_V \qquad (3)$$

The fuzzy connective AND in the (3) is implemented through the **t** – norm given by the product:

$$\mu_H = \mu_M \cdot \mu_V \qquad (4)$$

The low density indicator μ_L is constructed in a similar way, using the expression "not to high and not to low" in place of "high" in the sentence p_V. This kind of modification of the meaning of the sentence have been obtained suing standard fuzzy modifiers.

First simulation: time evolution of the population
The first simulation shows a considerable expansion of the pre-existing towns located around the main city together with the birth of two news towns and some new areas of scattered urbanisation. The river seems to drive a diffused development on both their banks. The birth of the new town at the tops of the picture can be explained considering that in a small area near the new town several attracting elements like railway, motorway and river cross one another.

Second simulation: time evolution of the indicator μ_H
The simulation enable a deeper inspection into the model of urbanisation of the area. The analysis of the level of centrality shows that the diffuse urbanisation is indeed characterized by the presence of some distinguished area of aggregation breaking the apparent uniformity of the wide urbanising zone along the river. A distinguished small zone on the left and right bank near the main city shows a slight tendency to evolve into a more important centre. This is connected to the fact that this is an advantaged position where the attractiveness of the near city, the low local concentration of activities and the presence of the river cooperate to give a high quality zone.

Third simulation: time evolution of the indicator μ_L
This simulation add the most interesting piece of information to the whole picture: each town tends to develop a small centre with a related peripheral area. The picture emerging from this simulations show a certain tendency of the urban system towards a polycentricity, though no centre emerges able to compete with the one of the main city. It is worthwhile to notice that a suitable set of meaningful indicators is needed in order to extract relevant information about the typology of the growth and urbanisation process.